Progress on Difference Equations
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Book of Abstracts

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Covilhã, Portugal

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Abstracts of Invited Talks

Recent Results and Improvements of Dynamic Opial-Type Inequalities

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Abstract

We give an overview of classical Opial inequalities both in the continuous and the discrete case, then extend the inequality to the dynamic case on time scales. We give many extensions of this inequality, among them extensions to the weighted case with one or two weights and the case involving higher-order derivatives. We also offer improvements of these inequalities, so-called Shum-type inequalities on time scales. Some applications contain the study of disconjugacy and disfocality of second-order dynamic equations on time scales. Nabla and diamond-alpha versions of the presented inequalities will be discussed as well.

Towards a theory of global dynamics in difference equations: Application to population dynamics

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Abstract

Global dynamics of difference equations/discrete dynamical systems are the most challenging problems in these disciplines. In this talk, we will explore some of the recent breakthroughs and advances in this area. The global dynamics of two types of discrete systems (maps) have been successfully established. These are triangular difference equations (maps) and monotone discrete dynamical systems (maps). We establish a general dynamical theory of triangular maps with minimal conditions. Smiths theory of planar monotone discrete dynamical systems is generalized via a new geometric theory to any finite dimension. Then we show how to establish global dynamics for maps that are neither monotone nor triangular via singularity theory and the notion of critical curves.

Applications to hierarchical competition models as well as predator-prey models will be discussed.

(q; h)–Weyl Algebras

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Abstract

In this talk I shall present $(q; h)$–deformed Weyl algebra and its properties and representations, show the connection to the qdeformed universal enveloping algebra and discuss the factorisation of the $(q; h)$–difference equations.

References

[1] S. Hilger, G. Filipuk, Factorization of $(q; h)$–difference operators – an algebraic approach, *J. Difference Eq. Appl.* 20 (8) (2014), 1201–1221.

[2] S. Hilger, G. Filipuk, R. Kycia, A. Dobrogowska, On the $(q; h)$–discretization of ladder operators, *International Journal of Difference Equations* 9 (1) (2014), 67–76.

[3] G. Filipuk, S. Hilger, Linear graininess time scales and ladder operators of orthogonal polynomials, *Results in Mathematics* 64 (2013), 13–35.

[4] S. Hilger, G. Filipuk, Algebra embedding of $U_q(sl(2))$ into the tensor product of two $(q; h)$–Weyl algebras, submitted.

[5] G. Filipuk, S. Hilger, A remark on the tensor product of two $(q; h)$–Weyl algebras, submitted.

Non-integrability of some difference equations

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Abstract

We consider the problem of characterizing, for certain natural number m, the local $\mathcal{C}^m$-non-integrability near elliptic fixed points of smooth planar measure preserving maps. Our criterion relates this non-integrability with the existence of some Lie Symmetries associated to the maps, together with the study of the finiteness of its periodic points. Our main result is:

Theorem *Let F be a $\mathcal{C}^{2n+2}$-planar map defined on an open set $\mathcal{U} \subseteq \mathbb{R}^2$ with an elliptic fixed point p, not $(2n+1)$-resonant, and such that its first non-vanishing Birkhoff constant is $B_n = i\, b_n$, for some $0 < n \in \mathbb{N}$ and $b_n \in \mathbb{R} \setminus \{0\}$. Moreover, assume that F is a measure preserving map with a non-vanishing density $\nu \in \mathcal{C}^{2n+3}$. If, for an unbounded sequence of natural numbers $\{N_k\}_k$, F has finitely many N_k-periodic points in $\mathcal{U}$ then it is not $\mathcal{C}^{2n+4}$-locally integrable at p.*

This criterion can be applied to prove that the Cohen map

$$F(x,y) = \left(y, -x + \sqrt{y^2+1}\right),$$

is not $\mathcal{C}^6$-locally integrable at its fixed point. Similarly we obtain non-integrability results for rational maps of the forms

$$F(x,y) = \left(y, \frac{f(y)}{x}\right) \quad \text{and} \quad F(x,y) = (y, -x + f(y)). \tag{1}$$

Note that the above maps contain Lyness and McMillan-Gumowski-Mira type difference equations.

This talk is based on a joint work with Anna Cima and Víctor Mañosa.

Dynamics and vaccination games

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Abstract

In general, the vaccination risks are over valorized regarding the vaccination benefits. When vaccination is voluntary, we study people decisions with respect to vaccination. The decision of an individual is influenced by the morbidity risks from vaccination, but also by the morbidity risks from infection and by the decisions of all other people. In this work, we make a game theoretical analysis of people decisions and take a special emphasis on the effects of vaccine scares and the effects of education programs. We introduce the ODE for the dynamic evolution of an individual vaccination strategy and observe that the stable equilibria of the ODE are the evolutionary stable strategies.

Nonautonomous Dynamics at work: Analytical and numerical analysis of a population-dynamical model

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Abstract

We apply various numerical and analytical tools to obtain an insight into the local and global dynamics of a discrete-time planar model from population dynamics with aperiodic coefficients. Due to the lack of equilibria and the insignificance of eigenvalues, we employ numerical schemes to approximate entire solutions, their dichotomy spectra, as well as the corresponding invariant manifolds. Moreover, we aim to illustrate the existing bifurcation theory for nonautonomous difference equations.
This talk is based on a joint work with Thorsten Hüls.

References

[1] T. Hüls and C. Pötzsche. Qualitative analysis of a nonautonomous Beverton-Holt Ricker model. *SIAM J. Applied Dynamical Systems* **13(4)** (2014), 1442–1488

Abstracts of Contributed Talks

Anosov diffeomorphisms and tilings

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Abstract

We consider a toral Anosov automorphism $G : \mathbb{T} \to \mathbb{T}$ given by $G(x, y) = (ax + y; x)$, where $a > 1$ is a fixed integer, and introduce the notion of γ-tiling to prove the existence of a one-to-one correspondence between (i) smooth conjugacy classes of Anosov diffeomorphisms with invariant measure absolutely continuous with respect to the Lebesgue measure and topologically conjugate to G, (ii) affine classes of -tilings and (iii) solenoid functions. Solenoid functions provide a parametrization of the infinite dimensional space of the mathematical objects described in these equivalences. This talk is based on a joint work with Alberto Pinto

References

[1] Pinto, A. A. and Almeida, J. P. : Anosov dffeomorphisms and γ-tilings. (*submitted*)
[2] Pinto, A. A., Almeida, J. P. and Portela, A.: Golden tilings. *Transactions of American Mathematical Society*, 364 (2012) 2261-2280.
[3] Pinto, A. A., Rand, D. A., Ferreira, F.: *Fine structures of hyperbolic diffeomorphisms*, Springer Monograph in Mathematics, Springer Verlag (2009).
[4] Pinto, A. A. and Rand, D. A.: Teichmüller spaces and HR structures for hyperbolic surface dynamics. *Ergod. Th. Dynam. Sys.* 22, 1905–1931 (2002).

[1] This work was supported by the FCT - Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology) within project UID/EEA/50014/2013 and ERDF - European Regional Development Fund through the COMPETE Program (operational program for competitiveness) and by National Funds through the FCT within Project “Dynamics and Applications” with reference PTDC/MAT/121107/2010.

On the solutions of a discrete Schrödinger equation

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Abstract

This talk is concerned with the asymptotic behavior of solutions of the scalar second-order difference equations of the form

$$u_{n+1} + u_{n-1} - q_n u_n = 0, \tag{1}$$

initially studied by Stepin and Titov [1]. They derived the asymptotic behavior of solutions under the assumptions that $q_n > 0$ for all n and that certain summability conditions of products of q_n^{-1} such as $\sum_{n=1}^{\infty} (q_n q_{n-1})^{-1} < \infty$ hold. In [2], we took a different approach to the study of this class of difference equations, which allowed us us to extend some of their results.

Related results also apply to

$$w_{n+1} + a_n w_{n-1} - b_n w_n = 0, \qquad n \geq 0,$$

which can be reduced to (1). Here a_n and b_n are supposed to be positive and to satisfy related summability conditions.

This talk is based on joint work with D.A. Lutz from San Diego State University.

References

[1] S.A. Stepin and V.A. Titov: Dichotomy of WKB–solutions of discrete Schrödinger equation, *Journal of Dynamical and Control Systems*, Vol. 12 (2006), pp. 135–144.

[2] S. Bodine and D.A. Lutz, *Asymptotic Integration of Differential and Difference Equations*, Springer, *Lecture Notes in Mathematics*, New York, 2015.

A note on the onset of synchrony in avian ovulation cycles

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Abstract

Spontaneous oscillator synchrony occurs when populations of interacting oscillators begin cycling together in the absence of environmental forcing. Synchrony has been documented in many physical and biological systems, including estrous/menstrual cycles in rats and humans. In previous work we showed that Glaucous-winged Gulls (*Larus glaucescens*) can lay eggs synchronously on an every-other-day schedule, and that synchrony increases with colony density. Here we pose a discrete-time model of avian ovulation to study the dynamics of synchronization. We prove the existence and uniqueness of an equilibrium solution which bifurcates to increasingly synchronous cycles as colony density increases.
This talk is based on joint work with Shandelle M. Henson, Professor of Mathematics and Chair Department of Mathematics, Andrews University, MI, USA.

References

[1] D. Burton and S. M. Henson, A note on the onset of synchrony in avian ovulation cycles, *Journal of Difference Equations and Applications*, Vol. 20, No. 4 (2014), pp. 664-668.

[1]This work was partial supported by the National Science Foundation (DMS-0613899) and by Andrews University Faculty Grants

Effects of treatment, awareness and condom use in a coinfection model for HIV and HCV in MSM

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Abstract

We develop a new a coinfection model for hepatitis C virus (HCV) and the human immunodefficiency virus (HIV). We consider treatment for both diseases, screening, unawareness and awareness of HIV infection, and the use of condom. We study the local stability of the disease-free equilibria for the full model and for the two submodels (HCV only and HIV only submodels). We sketch bifurcation diagrams for different parameters, such as the probabilities that a contact will result in a HIV or an HCV infection. We present numerical simulations of the full model where the HIV, HCV and double endemic equilibria can be observed. We also show numerically the qualitative changes of the dynamical behavior of the full model for variation of relevant parameters. We extrapolate the results from the model for actual measures that could be implemented in order to reduce the number of infected individuals.

[1]This work was partial supported by a FCT grant with reference SFRH/BD/96816/2013

On para-orthogonal polynomials on the unit circle and related questions

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Abstract

Our goal is to provide the para-orthogonality [1] theory, in the context of moment linear functional, with recurrence relation and the analog result to the classical Favard Theorem or Spectral Theorem.
This talk is based on a joint work with R. Cruz-Barroso and F. Perdomo-Pío.

References

[1] W. B. Jones, O. Njåstad, and W. J. Thron. Moment theory, orthogonal polynomials, quadrature, and continued fractions associated with the unit circle. Bull. London Math. Soc., 21: 113–152, 1989.

[1]This work was partial supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico of Brazil and Dirección General de Investigación, Ministerio de Economía y Competitividad of Spain under grant MTM2012-36732-C03-01.

Finding Invariant Fibrations for some Birational Maps of $\mathbb{C}^2$

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Abstract

Consider the family of fractional maps

$$f(x,y) = \left(\alpha_0 + \alpha_1 x + \alpha_2 y, \frac{\beta_0 + \beta_1 x + \beta_2 y}{\gamma_0 + \gamma_1 x + \gamma_2 y}\right), \tag{1}$$

where the parameters are complex numbers. In the paper "Dynamical classification of a Family of Birational Maps of $\mathbb{C}^2$ via algebraic entropy", see [1], the authors find the *algebraic entropy* of such maps depending on the parameters.
The algebraic entropy is defined as the logarithm of the *dynamical degree*, $\delta(F)$, which in turn is defined as

$$\delta(F) := \lim_{n\to\infty} (\deg(F^n))^{\frac{1}{n}}. \tag{2}$$

Birational maps with zero algebraic entropy are classified in three types, depending on the growth rate of the sequence of degrees, see [2]:

- The sequence of degrees grows linearly and f preserves a unique fibration given by curves of genus zero.
- The sequence of degrees grows quadratically and f preserves a unique fibration given by curves of genus one.
- The sequence of degrees is bounded and f preserves two fibrations generically transverse.

In this talk I'm going to explain how we find the invariant fibrations of some maps of the form (1).
This talk is based on a joint work with Sundus Zafar.

References

[1] Cima A. and Zafar, S., Dynamical classification of a Family of Birational Maps of $\mathbb{C}^2$ via Algebraic Entropy, Preprint, 2014.

[2] Diller, J. and Favre, C. , Dynamics of bimeromorphic maps of surfaces, *Amer. J. Math.*, Vol.123, (2001), pp. 1135–1169.

Factorization method applied to the second order q-difference operators

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Abstract

We present certain classes of second order q-difference operators, which admit factorization into first order operators acting in a Hilbert space. By solving an infinite nonlinear system of q-difference equations one constructs a chain of q-difference operators. The eigenproblems for the chain are solved and some applications, including the ones related to q-Hahn orthogonal polynomials, $q-$ Morse potential, are discussed. We also discuss classical limit case by letting $q \to 1$. It is shown that in the limit the present method corresponds to the one developed by Infeld and Hull.

References

[1] A. Dobrogowska, A. Odzijewicz, Second order q-difference equations solvable by factorization method. *J. Comput. and Appl. Math.*, 193:319–346, 2006.
[2] A. Dobrogowska, A. Odzijewicz, Solutions of the q-deformed Schrrödinger equation for special potentials, *J. Phys. A: Math. Theor.*, 40:2023–2036, 2007.
[3] A. Dobrogowska, The q-deformation of the Morse potential, *Appl. Math. Lett.*, 7:769–773, 2013.
[4] A. Dobrogowska, G. Jakimowicz, Factorization method applied to the second order q-difference operators, *Appl. Math. Comput.*, 228:147-152, 2014.
[5] L. Infeld, T.E. Hull, The Factorization Method, *Rev. Mod. Phys.*, 23:21-68, 1951.

The numerical solutions for a system of fuzzy differential equations by fuzzy Laplace transform method

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Abstract

The fuzzy Laplace transform algorithm is one of the appropriate methods to study linear or nonlinear differential problems, in both crisp and fuzzy cases.
In this paper, we use the fuzzy Laplace transform method, introduced by the authors in [2], to calculate exact solutions of a system of fuzzy linear differential equations, under generalized Hukuhara differentiability, using respectively Hukuhara and Minkowski differences. Then, we establish a comparison between the intervals of obtained solutions.
This talk is based on a joint work with S.MELLIANI and L.S. CHADLI.

References

[1] S. Abbasbandy, T. Allahviranloo, O. Lopez-Pouso and J.J.Nieto, Numerical methods for fuzzy differential inclusions, *Comput.Math.Appl.*, Vol. 48 (2004), pp. 1633–1641.
[2] T. Allahviranloo , M. B. Ahmadi, Fuzzy Laplace transforms, *Soft Computing*, Vol. 14 (2010), pp. 235-243.
[3] T. Allahviranloo , S. Abbasbandy, S. Salahshour and A. Hakimzadeh, A new method for solving fuzzy linear differential equations, *Computing*, Vol. 92 (2011), pp. 181–197.
[4] T. Allahviranloo , N. Ahmady and E. Ahmady, Numerical solution of fuzzy differential equations by predictor-corrector method, *Information Sciences* Vol. 177 (2007), pp. 1633–1647.
[5] T. Allahviranloo, N. A. Kiani and N. Motamedi, Solving fuzzy differential equations by differential transformation method, *Information Sciences* Vol. 179 (2009), pp. 956–966.
[6] R. J. Aumann, Integrals of set-valued functions, *J. Math. Anal. Appl.*, Vol. 12 (1965), pp. 1–12.
[7] B. Bede and S. G. Gal, Generalization of the Differentiability of Fuzzy-Number-Valued Functions with Applications to Fuzzy Differentiel Equations, *Fuzzy Sets and Systems*, Vol. 151 (2005), pp. 581–599.
[8] B. Bede, I. J. Rudas and A. L. Bencsik, First order linear fuzzy differential equations under generalized differentiability, *Inf. Sci.*, Vol. 177 (2006), pp.1648–1662.
[9] Y. Chalco-Cano and H. Roman-Flores, On new solutions of fuzzy differential equations, *Chaos Solitons Fractals*, Vol. 38 (2006), pp. 112-119.

[10] E. Eljaoui, S. Melliani and L.S. Chadli, Solving second-order fuzzy differential equations by the fuzzy Laplace transform method, *Advances in Difference Equations*, Vol. 66 (2015), pp. 1–14.
[11] O. Kaleva, Fuzzy Differentiel Equations, *Fuzzy Sets and Systems*, Vol. 24 (1987), pp. 301–317.
[12] O. Kaleva, The Cauchy Problem for Fuzzy Differentiel Equations, *Fuzzy Sets and Systems*, Vol. 35 (1990), pp. 366–389.
[13] S. Melliani, E. Ejaoui and L.S. Chadli, Solving fuzzy linear differential equations by a new method, *Annals of Fuzzy Mathematics and Informatics*, Vol. 9, No. 2 (2015), pp. 307–323.
[14] J. J. Nieto, R. Rodriguez-Lopez and D. Franco, Linear first-order fuzzy differential equations, *Int. J. Uncertain Fuzziness Knowledge-Based Syst.*, Vol. 14 (2006), pp. 687–709.
[15] D. O'Regan, V. Lakshmikantham and J. J. Nieto, Initial and Boundary Value Problems for Fuzzy Differentiel Equations, *Nonlinear Analysis*, Vol. 54 (2003), pp. 405–415.
[16] M. L. Puri, D. A. Ralescu, Fuzzy random variables, *J. Math. Anal. Appl.*, Vol. 114 (1986), pp. 409–422.
[17] S. Salahshour and T. Allahviranloo, Applications of fuzzy Laplace transforms, *Soft Comput*, Vol. 17 (2013), pp. 145158
[18] S. Salahshour, M. Khezerloo , S. Hajighasemi and M. Khorasany, Solving Fuzzy Integral Equations of the Second Kind by Fuzzy Laplace Transform Method, *Int. J. Industrial Mathematics*, Vol. 4, No. 1 (2012), pp. 21–29.
[19] M. Thongmoona, S. Pusjuso, The numerical solutions of differential transform method and the Laplace transform method for a system of differential equations, *Nonlinear Analysis: Hybrid Systems*, Vol. 4 (2010), pp. 425–431.
[20] H.C. Wu, The improper fuzzy Riemann integral and its numerical integration, *Inform. Sci.*, Vol. 111 (1999), pp. 109-137.
[21] H.C. Wu, The fuzzy Riemann integral and its numerical integration, *Fuzzy Set Syst.*, Vol. 110 (2000), pp. 1-25.

Practical invariant polyhedral sets of discrete-time fractional systems

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Abstract

During the talk a new notion of the practical invariance (viability) of fractional discrete-time linear and nonlinear systems is introduced. The invariance is considered on polyhedral sets. Sufficient conditions for both, linear and nonlinear fractional systems, providing practical invariancy are given.
This talk is based on a joint work with Dorota Mozyrska.

[1]This work was supported by Białystok University of Technology grants G/WM/3/12 and S/WI/02/2011.

Systems of difference equations with solutions associated to Fibonacci numbers

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Abstract

This paper deal with form, the periodicity and the stability of the solutions of the systems of difference equations

$$x_{n+1} = \frac{1}{1+y_{n-2}},\ y_{n+1} = \frac{1}{1+x_{n-2}},\ n, \in \mathbb{N}_0,$$

where $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$ and the initial conditions x_{-2}, x_{-1}, x_0, y_{-2}, y_{-1}, and y_0 are non zero real numbers.

References

[1] Elsayed E. M, O*n the Solutions of Some Systems of Second Order Rational Difference Equations*, Life Science Journal, (3) (2013), 344-351.

[2] Elsayed E. M, *Solutions of some rational systems of difference equations*, Utilitas Mathematica, **6** (2013), 329-336.

[3] Elsayed E. M, *On the solutions of some systems of difference equations*, Archives Des Sciences, **5** (2013), 369-378.

[4] Halim Y, *Global character of systems of rational difference equations*, Electronic Journal of Mathematical Analysis and Applications, **3**(1) 2015, 204-214.

[5] Touafek N, Halim Y, *Global attractivity of a rational difference equation*, Mathematical Sciences Letters, **2** (2013), 161-165.

[6] Touafek N, Halim Y, *On Max Type Difference Equations: Expressions of Solutions*, International Journal of Nonlinear Science, **11** (2011), 396-402.

[7] Yazlik Y, Tollu D. T and Taskara N, *On the solutions of difference equation systems with Padovan numbers*, Applied Mathematics, **4** (12A) (2013), 15-20.

[1]This work was partial supported by LMAM laboratory, Jijel university, Algeria

The two versions of the weakest form of distributional chaos

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Abstract

The notion of distributional chaos was introduced by Schweizer and Smítal in [1] for continuous maps of the interval. However, it turns out that, for continuous maps of a compact metric space three mutually nonequivalent versions of distributional chaos, DC1 – DC3, can be considered.
The talk will be devoted to the recent results concernig the weakest form of distributional chaos (denoted by DC3). We show that in a general compact metric space, distributional chaos of type 3, even when assuming existence of an uncountable scrambled set, is a very weak form of chaos. In particular, (i) the chaos can be unstable (it can be destroyed by conjugacy), and (ii) such an unstable system may contain no Li-Yorke pair. However, definition can be strengthened to get DC$2\frac{1}{2}$ which is topological invariant and implies Li-Yorke chaos, similarly as types DC1 and DC2, but unlike them, strict DC$2\frac{1}{2}$ systems must have zero topological entropy.

References

[1] B. Schweizer, J. Smítal Measures of chaos and a spectral decomposition of dynamical systems on the interval, *Trans. Amer. Math. Soc.*, Vol. 344 (1994), pp. 737–754.

On Mittag–Leffler stability of fractional dynamic equations on time scales

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Abstract

In this study we focus on establishing sufficient conditions for Mittag–Leffler stability and global stability. We give stability conditions for the solutions of both Caputo type and Riemann–Liouville type fractional order delta dynamic equations on time scales.
This talk is based on a joint work with Deniz Uçar.

References

[1] A. A. Kilbas, H. M. Sirvastava, J. J. Trujillo,*Theory and Applications of Fractional Differential Equations*, Elsevier B.V., 2006.
[2] A. Ahmadkhanlu, M. Jahanshahi, On the existence and uniqueness of solution of initial value problem for fractional order differential equations on time scales, *Bulletin of the Iranian Mathematical Society,* Vol. 38 No. 1 (2012), pp 241–252.
[3] C. Yakar, Fractional Differential Equations in Terms of Comparison Results and Lyapunov Stability with Initial Time Difference, *Abstract and Applied Analysis,* Vol. 2010, Article ID 762857, (2010), pp. 16.
[4] F. Chen, Fixed points and asymptotic stability of nonlinear fractional difference equations, *Electronic Journal of Qualitative Theory of Differential Equations,* No. 39, (2011), pp. 1–18.
[5] F. Chen, Z. Liu, Asymptotic stability results for nonlinear fractional difference equations, *Journal of Applied Mathematics*, Vol. 2012, Article ID 879657, (2012), pp. 14.
[6] F. Jarad, T. Abdeljawad, D. Baleanu, K. Biçen, On the stability of some discrete fractional nonautonomous systems. *Abstract and Applied Analysis,* Vol. 2012, Article ID 476581, (2012), pp. 9.
[7] F, Jarad, T. Abdeljawad, E. Gundogdu, D. Baleanu, On the Mittag–Leffler stability of q–fractional nonlinear dynamical systems. *Proc. Romanian Acad. A,* Vol. 12 No. 4, (2011), pp. 309–314.
[8] F. M. Atici, P. W. Eloe, A transform method in discrete fractional calculus, *International Journal of Difference Equations*, Vol. 2, No. 2, (2007), pp. 165–176.
[9] F.M.Atıcı, P.W.Eloe, Fractional q–calculus on a time scale, *J. Nonlinear Math. Phys.*, Vol. 14, No. 3, (2007), pp. 333–344.
[10] F.M.Atıcı, P.W.Eloe, Linear systems of fractional nabla difference equations, *Rocky Mountain J.Math.,* Vol. 41, No. 2 (2011), pp. 353–370.
[11] G. Samko, A. A. Kilbas, and O. I. Marichev, *Fractional Integrals and Derivatives: Theory and Applications*, Gordon and Breach Science, Yverdon, 1993.

[12] G. A. Anastassiou, Foundations of nabla fractional calculus on time scales and inequalities, *Computers & Mathematics with Applications,* Vol. 59, No. 12, (2010), pp. 3750–3762.

[13] G. A. Anastassiou, Principles of delta fractional calculus on time scales and inequalities, *Mathematical and Computer Modelling*, Vol. 52, No. 3–4, (2010), pp. 556–566.

[14] I. Podlubny, *Fractional Differential Equations*, Academic Press, San Diego, 1999.

[15] J.C. Trigeassou, N. Maamri, J. Sabatier, A. Oustaloup, A Lyapunov approach to the stability of fractional differential equations, *Signal Processing*, Vol. 91, No. 3, (2011), pp. 437–445.

[16] J. M. Davis, I. A. Gravagne, B. J. Jackson, R. J. Marks, and A. A. Ramos, The Laplace transform on time scales revisited, *Journal of Mathematical Analysis and Applications,* Vol. 332, No. 2, (2007), pp. 1291–1307.

[17] J. Zhu, Y. Zhu, Fractional Cauchy Problem with Riemann–Liouville Fractional Delta Derivative on Time Scales, *Abstract and Applied Analysis,* Vol. 2013, Article ID 401596, (2013), pp.19.

[18] K. S. Miller and B. Ross, *An introduction to the fractional calculus and fractional differential equations*, A Wiley–Interscience Publication. John Wiley &Sons Inc., NewYork, 1993.

[19] M. Bohner, A. Peterson, *Advances in dynamic equations on time scales*, Birkhauser, Boston, 2003.

[20] M. Bohner, A. Peterson, *Dynamic equations on time scales an introduction with applications*, Birkhauser, Boston, 2001.

[21] M. Bohner, G.S. Guseinov, The convolution on time scales, *Abstract and Applied Analysis,* Vol. 2007, Article ID 58373, (2007), pp. 24.

[22] M. Bohner, G.S. Guseinov, B. Karpuz, Properties of the Laplace transform on time scales with arbitrary graininess, *Integral Transforms and Special Functions,* Vol. 22, No. 11, (2011), pp. 785–800.

[23] M. Wyrwas, D. Mozyrska, *On Mittag–Leffler stability of fractional order difference systems*, Advances in Modelling and Control of Non–integer–Order Systems, Springer International Publishing,.2015.

[24] N. Benkhettou, A.M.C. Brito da Cruz, D. F.M. Torres, A fractional calculus on arbitrary time scales: Fractional differentiation and fractional integration, *Signal Processing,* Vol. 107, (2015), pp. 230–237.

[25] N.R.O. Bastos, *Fractional calculus on time scales [Ph.D. thesis],* The University of Aveiro, 2012.

[26] N.R.O. Bastos, D.Mozyrska, D.F.M.Torres, Fractional derivatives and integrals on time scales via the inverse generalized Laplace transform, *Int.J.Math.Comput.,* Vol.11, No. J11 (2011), pp. 1–9.

[27] P. A. Williams, *Unifying fractional calculus with time scales [Ph.D. thesis]*, The University of Melbourne, 2012.

[28] S. J. Sadati, D. Baleanu, A. Ranjbar, R. Ghaderi, T. Abdeljawad (Maraaba), Mittag–Leffler stability theorem for fractional nonlinear systems with delay, *Abstract and Applied Analysis,* Vol. 2010, Article ID 108651, (2010), pp.7.

[29] T. Abdeljawad, On Riemann and Caputo fractional differences, *Computers & Mathematics with Applications,* Vol. 62, No. 3, (2011), pp. 1602–1611.

[30] Y. Li, Y. Q. Chen, I. Podlubny, Mittag–Leffler stability of fractional order nonlinear dynamic systems, *Automatica,* Vol. 45, (2009), pp. 1965–1969.

[31] Y. Li, Y. Q. Chen, I. Podlubny, Stability of fractional–order nonlinear dynamic systems: Lyapunov direct method and generalized Mittag–Leffler stability, *Comput. Math. Appl.*,Vol. 59, (2010), pp. 1810–1821.

The generating function of the generalized Fibonacci sequence

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Abstract

Using tools of the theory of orthogonal polynomials and somes results in [1], we obtain the generating function of the generalized Fibonacci sequence established in [2] for a sequence of real or complex numbers $\{Q_n\}_{n=0}^{\infty}$ defined by

$$Q_0 = 0,\ Q_1 = 1,\ Q_m = a_j Q_{m-1} + b_j Q_{m-2},\ m \equiv j(mod\ k),$$

where $k \geq 3$ is a fixed integer, and $a_0, a_1, \ldots, a_{k-1}$, $b_0, b_1, \ldots, b_{k-1}$ are $2k$ given real or complex numbers, with $b_j \neq 0$ for $0 \leq j \leq k-1$. For this sequence some convergence proprieties are obtained.
This talk is based on a joint work with Armando Gonçalves (Department of Mathematics, University of Coimbra, Portugal).

References

[1] M. N. de Jesus and J. Petronilho, On orthogonal polynomials obtained via polynomial mappings, *J. Approx. Theory*, **162** (2010) 2243-2277.
[2] J. Petronilho, *Generalized Fibonacci sequences via orthogonal polynomials*, Appl. Math. Comput. **218** (2012), 9819-9824.

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On the Convergence of Exponential Expanding Meshes for Singular Poisson Equation

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Abstract

We develop a third (fourth) order accurate new finite difference scheme for the numerical solution of the Poisson equation in polar coordinate. The method is refined in such a manner so that it is applicable to both singular and non-singular cases, while order and accuracy being unchanged. The special character of the geometric mesh ratio parameter will take care of interior or boundary layers, if any. A detailed convergence theory for the difference scheme has been developed based on monotone and irreducible property of the iteration matrices. Numerical accuracy of the solutions has been obtained that shows the applicability of the scheme in the presence of singularity and thin layers.

References

[1] D. Britz, *Digital simulation in electrochemistry*, Springer, Berlin, Heidelberg, 2005.
[2] R.K. Mohanty, Order h^4 difference methods for a class of singular two space elliptic boundary value problems, *Journal of Computational and Applied Mathematics*, Vol. 81, (1997), pp. 229–247.

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Global Stability and Periodic Solutions of a Second Order Rational Equation with Applications

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Abstract

We investigate the boundedness and persistence of solutions, the global stability of the positive fixed point and the occurrence of periodic solutions for the rational difference equation

$$x_{n+1} = ax_n + bx_{n-1} + \frac{\alpha x_n + \beta x_{n-1} + \gamma}{Ax_n + Bx_{n-1} + C}$$

with non-negative parameters and initial values. We establish that when the function defining the difference equation is monotone in its arguments, the equation does not have any periodic solutions of period greater than two and in the absence of two-cycles, the solutions converge to the unique positive fixed point. In addition, we show that the above results can be used in the study of several distinct classes of planar systems.
This talk is based on a joint work with Hassan Sedaghat.

References

[1] M. Dehghan, C.M. Kent, R. Mazrooei-Sebdani, N.L.Ortiz, H. Sedaghat, Dynamics of rational difference equations containing quadratic terms, *J. Difference Eq. Appl.*, Vol. 14, No. 2 (2008), pp. 191-208.

[2] M. Dehghan, C. M. Kent, R. Mazrooei-Sebdani, N.L. Ortiz, H. Sedaghat, Monotone and oscillatory solutions of a rational difference equation containing quadratic terms, *J. Difference Eq. Appl.*,Vol. 14, Nos.10-11 (2008), pp. 1045-1058.

[3] M.R.S. Kulenovic and G. Ladas, *Dynamics of second order rational difference equations with open problems and cojectures*, CRS Press, Boca Raton, 2002.

[4] H. Sedaghat, Folding, Cycles and Chaos in Planar Systems, *J. Difference Eq. Appl.*, Vol. 21, No. 1 (2015), pp. 1-15.

On quasi-periodicity properties of fractional sums and fractional differences of periodic functions

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Abstract

This talk is related with discrete fractional calculus [1]; our goal is to investigate quasi-periodic properties of fractional order sums and differences of periodic functions. Using Riemann-Liouville and Caputo type definitions, we study concepts close to the well known idea of periodic function, such as asymptotically periodicity or S-asymptotically periodicity. We use basic tools of discrete fractional calculus. Boundedness of sums and differences of fractional order of periodic functions are also investigated.

It is an obvious fact that the classical derivative, if it exists, of a periodic function is also a periodic function of the same period. Moreover, the primitive of a periodic function may be periodic. The same holds also for difference and sum operators. Nevertheless, when we consider derivatives or integrals of non integer order, this is not true (see, for example, [2, 3]). In [4], we have studied some properties of fractional integrals and derivatives of periodic functions.

Motivated by [4, 5], we present here the analogous results in the field of discrete fractional calculus. We also point out one important difference between continuum and discrete fractional operators.

References

[1] K.S. Miller and B. Ross, Fractional difference calculus, *International Symposium on Univalent Functions, Fractional Calculus and Their Applications*, Nihon University, Koriyama, 1988, pp. 139–142.

[2] M.S. Tavazoei, A Note on Fractional-Order Derivarives of Periodic Functions. *Automatica*, Vol. 46:5 (2010), pp. 945–948.

[3] I. Area, J. Losada and J.J. Nieto, On fractional derivatives and primitives of periodic functions, *Abstract and Applied Analysis*, Article ID 392598 (2014), 8 pp.

[4] I. Area, J. Losada and J. J. Nieto, On quasi-periodicity properties of fractional integrals and fractional derivatives of periodic functions, (Submitted).

[5] J. Diblík, M. Fečkan and M. Posipíšil, Nonexistence of periodic solutions and S-asymptotically periodic solutions in fractional difference equations, *Applied Mathematics and Computation*, Vol. 257 (2015), pp. 230-240.

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Krause's Model of Opinion Dynamics on Time Scales

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Abstract

We analyze bounded confidence models on time scales. In such models each agent takes into account only the assessments of the agents whose opinions are not too far away from his own opinion. We prove a convergence into clusters of agents, with all agents in the same cluster having the same opinion. The necessary condition for reaching a consensus is given. Simulations are performed to validate the theoretical results.
This talk is based on a joint work with E. Girejko, L. M. F. Machado and N. Martins.

Fronts and pulses that fail to propagate in discrete inhomogeneous media

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Abstract

Bistable differential-difference equations are often used to model the conduction of electrical impulses in the nervous system. One characteristic of diseases that affect the nervous system is that a portion of the medium for conduction is deteriorated, and it is particularly interesting to understand what causes electrical impulses to fail to propagate under these circumstances. This leads to the study of a second order difference equation that is semi-linear, inhomogeneous, and has boundary conditions at infinity. A thorough study of the exact solutions of this system provides necessary and sufficient conditions for fronts [1] and pulses [2] to fail to propagate due to inhomogeneities in the medium.
This talk is based on joint work with A.R. Humphries, J.M. Segal, and E.S. Van Vleck

References

[1] A.R. Humphries, B.E. Moore, and E.S. Van Vleck, Front Solutions for Bistable Differential-Difference Equations with Inhomogeneous Diffusion, *SIAM Journal on Applied Mathematics*, Vol. 71, No. 4, (2011), pp. 1374–1400.

[2] B.E. Moore and J.M. Segal, Stationary Bistable Pulses in Discrete Inhomogeneous Media, *Journal of Difference Equations and Applications*, Vol. 20, No. 1 (2014), pp. 1–23.

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Stability of higher orders of fractional difference systems

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Abstract

The problem of the stability of the Caputo– and Riemann-Liouville–type linear discrete-time systems with fractional positive orders is discussed. We present the method of reducing the fractional order of the considered systems by transforming them to the multi-order linear systems with the partial orders from the interval $(0, 1]$. For the constructed multi-order systems the conditions for the stability of the considered linear systems are formulated based on the $\mathcal{Z}$-transform, that is considered as an effective method for stability analysis of linear systems. Finally, the presented examples show the conditions for asymptotic stability for some classes of the fractional linear systems with positive orders.
This talk is based on a joint work with Małgorzata Wyrwas.

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Factorization method for second order functional equations

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Abstract

This talk is based on the joint paper [1] with Tomasz Goliński where we developped the theory of equations of the form

$$\alpha(x)\psi(\tau(x)) + \beta(x)\psi(x) + \gamma(x)\psi(\tau^{-1}(x)) = \lambda\psi(x)$$

generated by a bijection $\tau : X \to X$ of the real line subset $X \subset \mathbb{R}$.
The equations of this type can be regarded as an alternate discretization of the second order differential equations including Schrödinger equation and generalization of difference and q-difference equations. They also emerge from the change of variables in difference equations. On the other hand the functional equations by themselves are of interest and have many important applications. They have been investigated in many monographs and papers from the point of view of functional analysis, especially C^*-algebras methods. We hope that our approach can be applied also in the numerical analysis of some problems.

References

[1] T. Goliński and A. Odzijewicz, Factorization method for second order functional equaitons. *J. Comput. and Appl. Math.*, 176, 2005.
[2] M. Klimek, Extension of q-deformed analysis and q-deformed models of classical mechanics. *J. Phys. A: Math. Gen.*, 26, 1993.
[3] W. Hahn, Über Orthogonalpolynome die q-Differenzengleichungen genüngen. *Math. Nachr.*, 2:4–34, 1949.
[4] A. Odzijewicz, M. Horowski, and A. Tereszkiewicz, Integrable mulit-boson systems and orthogonal polynomials. *J. Phys. A: Math. Gen.*, 34:4353–4376, 2001.
[5] A. Dobrogowska, A. Odzijewicz, Second order q-difference equations solvable by factorization method. *J. Comput. and Appl. Math.*, 193:319–346, 2006.

Fractional differential equations with dependence on the Caputo–Katugampola derivative

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Abstract

We present a new type of fractional operator, the Caputo–Katugampola derivative. The Caputo and the Caputo–Hadamard fractional derivatives are special cases of this new operator. An existence and uniqueness theorem for a fractional Cauchy type problem, with dependence on the Caputo–Katugampola derivative, is proven. A decomposition formula for the Caputo–Katugampola derivative is obtained. This formula allows us to provide a simple numerical procedure to solve the fractional differential equation.
This talk is based on a joint work with Ricardo Almeida and Agnieszka B. Malinowska.

[1]This work was partially supported by the Warsaw School of Economics grant KAE/S/35/15.

Rate of uniform consistency for nonparametric of the conditional quantile estimate with functional variables in the functional single-index model

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Abstract

The main objective of this paper is to estimate non-parametrically the quantiles of a conditional distribution when the sample is considered as an α-mixing sequence. First of all, a kernel type estimator for the conditional cumulative distribution function (*cond-cdf*) is introduced. Afterwards, we give an estimation of the quantiles by inverting this estimated *cond-cdf*, the asymptotic properties are stated when the observations are linked with a single-index structure. The pointwise almost complete convergence and the uniform almost complete convergence (with rate) of the kernel estimate of this model are established. This approach can be applied in time series analysis. For that, the whole observed time series has to be split into a set of functional data, and the functional conditional quantile approach can be employed both in foreseeing and building confidence prediction bands.
This talk is based on a joint work with Amina Angelika Bouchentouf and Souad Mekkaoui

References

[1] A. Aït Saidi, F. Ferraty and R. Kassa, Single functional index model for a time series, *R. Roumaine Math. Pures et Appl.*, Vol. 50 (2005), pp. 321-330.
[2] A. A. Bouchentouf, T. Djebbouri, A. Rabhi and K. Sabri, Strong uniform consistency rates of some characteristics of the conditional distribution estimator in the functional single-index model, *Appl. Math. (Warsaw)*, Vol. 41, No. 4 (2014), pp. 301–322.
[3] F. Ferraty, A. Laksaci, A. Tadj and P. Vieu, Rate of uniform consistency for nonparametric estimates with functional variables, *J. Statist. Plann. and Inf.*, Vol. 140 (2010), pp. 335–352.
[4] F. Ferraty, A. Rabhi and P. Vieu, Conditional quantiles for functional dependent data with application to the climatic El Ninõ phenomenon, *Sankhyã: The Indian Journal of Statistics, Special Issue on Quantile Regression and Related Methods*, Vol. 67, No. 2 (2005), pp. 378–399.
[5] F. Ferraty and P. Vieu, *Nonparametric Functional Data Analysis: Theory and Practice*, Springer Series in Statistics, Springer, New York, 2006.

On Laguerre-Hahn affine Orthogonal Polynomials on the Unit Circle from matrix Sylvester equations

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Abstract

Laguerre-Hahn affine orthogonal polynomials on the unit circle are related to Carathéodory functions, F, that satisfy first order differential equations with polynomial coefficients $zAF' = CF + D$. Well-known families of these polynomials include the semi-classical orthogonal polynomials on the unit circle [2, 4, 6] as well as some of their perturbations, such as the ones studied in [1, 3, 5, 8].
In this talk one derives recurrences for the reflection coefficients of Laguerre-Hahn affine orthogonal polynomials on the unit circle, including a form of the discrete Painlevé equations dP_V. The technique is based on the knowledge of the first order differential equation for the Carathéodory function, combined with a re-interpretation, in the formalism of matrix Sylvester equations, of compatibility conditions for the differential systems satisfied by the polynomials.

References

[1] A. Branquinho and M.N. Rebocho, *Difference and differential equations for deformed Laguerre-Hahn orthogonal polynomials on the unit circle*, J. Phys. A: Math. Theor. 44 (2011), no. 46, 465204 (12 pp).

[2] A. Branquinho and M.N. Rebocho, *Distributional equation for Laguerre-Hahn functionals on the unit circle*, J. Comput. Appl. Math. 233 (2009), pp. 634-642.

[3] A. Cachafeiro, F. Marcellán, and C. Pérez, *Lebesgue perturbation of a quasi-definite Hermitian functional. The positive-definite case*, Linear Algebra Appl. 369 (2003), pp. 235–250.

[4] A. Cachafeiro and C. Pérez, *A study of the Laguerre-Hahn affine functionals on the unit circle*, J. Comput. Anal. Appl. 6 (2) (2004), pp. 107-123.

[5] K. Castillo, L. Garza, and F. Marcellán, *Perturbations on the subdiagonals of Toeplitz matrices, Linear Algebra Appl.* 434 (2011), pp. 1563-1579.

[6] A. P. Magnus, MAPA3072A Special topics in approximation theory 1999-2000: Semi-classical orthogonal polynomials on the unit circle, http://www.math.ucl.ac.be/ magnus/.

[7] M. N. Rebocho, *On Laguerre-Hahn affine Orthogonal Polynomials on the Unit Circle from matrix Sylvester equations*, (submitted).

[8] N.S. Witte, *Bi-orthogonal systems on the unit circle, regular semi-classical weights and integrable systems - II*, J. Approx. Theory 161 (2009), pp. 565-616.

On a system of difference equations including negative exponential terms

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Abstract

In this paper we study the asymptotic behavior of the positive solutions of the system of two difference equations

$$x_{n+1} = ay_n + bx_{n-1}e^{-y_n}, \quad y_{n+1} = cx_n + dy_{n-1}e^{-x_n}, \quad n = 0, 1, \ldots$$

where a, b, c, d are positive constants and the initial values x_{-1}, x_0, y_{-1}, y_0 are positive numbers.
This talk is based on a joint work with Prof. G. Papaschinopoulos and Dr. N. Fotiades

References

[1] S. Elaydi, *An introduction to Difference Equations*, Springer-Verlag, New York, 1996.
[2] V. L. Kocic and G. Ladas, *Global behavior of nonlinear difference equations of higher order with applications*, Kluwer Academic Publishers, Dordrecht, 1993.
[3] E. A. Grove, G. Ladas, N.R. Prokup and R. Levis, On the global behavior of solutions of a biological model, *Communications on Applied Nonlinear Analysis*, 7 No. 2 (2000), 33-46.
[4] C. M. Kent, Converegence of solutions in a nonhyperbolic case, *Nonlinear Anal.*, 47, 2001, 4651-4665.
[5] S. Stević, n some solvable systems of difference equations, *Appl. Math. Comput.*, 218 (2012), 5010-5018.

Properties of solutions of system of difference equations

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Abstract

The purpose of this talk is to present some results concerning k-dimensional system of neutral difference equations with delays in the following form

$$\begin{cases} \Delta\Big(x_i(n) + p_i(n)\, x_i(n-\tau_i)\Big) = a_i(n)\, f_i(x_{i+1}(n-\sigma_i)) + g_i(n) \\ \Delta\Big(x_k(n) + p_k(n)\, x_k(n-\tau_k)\Big) = a_k(n)\, f_k(x_1(n-\sigma_k)) + g_k(n), \end{cases}$$

where $i = 1, \ldots, k-1$. Sufficient conditions for the existence of nonoscillatory bounded solutions of the above system with various $(p_i(n))$, $i = 1, \ldots, k$, $k \geq 2$ will be presented.
Joint work with Małgorzata Migda and Małgorzata Zdanowicz.

Bifurcation skeleton of a family of non autonomous systems generated by stunted sawtooth maps

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Abstract

Families of stunted sawtooth maps have been used as models to study related families of differentiable maps, since they are closely related with symbolic dynamics and are rich enough to encompass in a canonical way all possible kneading data and all possible itineraries. On the other hand, considered as gap maps, they have been used in the applications as models of systems of communication with chaos. In this talk we will consider a family of 2-periodic non autonomous dynamical systems, generated by the alternate iteration of two stunted sawtooth tent maps and study its bifurcation skeleton. We will describe the bifurcation phenomena along and around the bones accomplished with the combinatorial data furnished by the respective symbolic dynamics.
This talk is based on a joint work with Teresa Silva and J. Leonel Rocha.

Universal convergence rates for a non autonomous family of dynamical systems generated by stunted sawtooth maps

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Abstract

We study a family of non autonomous systems with period 2, generated by the sequential iteration of two *stunted sawtooth* maps. Using the concepts of symbolic dynamics, renormalization and star product in the non autonomous setting, we compute the convergence rates of sequences of points in the parameter space. These sequences are obtained through consecutive star products/renormalizations, generalizing in this way the Feigenbaum sequences. We show that the convergence rates are independent of the initial point, thus, concluding that the non autonomous setting has universal properties of the type founded by Feigenbaum in families of autonomous systems.
This talk is based on a joint work with Luís Silva, CIMA–UE, Évora, Portugal, and Sara Fernandes, Department of Mathematics, Universidade de Évora, Rua Romão Ramalho, 59, 7000-671 Évora, Portugal.

Non-standard finite difference method and application to the prey-predator model of fractional order

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Abstract

We present the non-standard finite difference method of the Mickens type. The essence of the non-standard difference schemes is preserving the significant properties of the original problems and/or their solutions such as, for example, positivity, monotonicity, boundedness, existence and stability of equilibrium points, etc. We construct the fractional non-standard finite difference scheme which preserves positivity and we analyze the convergence of the method. Finally, we use this method in the numerical analysis of the stability of solutions of the fractional prey-predator model to its equilibrium points..
This talk is based on a joint work with Jacky Cresson (Laboratoire de Mathématiques Appliquées de Pau, Université de Pau et des Pays de l'Adour, Pau Cedex, France).

References

[1] Ahmed E., El-Sayed A.M.A., El-Saka H.A.A., *Equilibrium points, stability and numerical solutions of fractional-order predatorprey and rabies models*, J. Math. Anal. Appl., Vol. 325 (2007) 542553.
[2] Javidi M., Nyamoradi N., *Dynamic analysis of a fractional order preypredator interaction with harvesting*, App. Math. Model. 37 (2013) 8946-8956.
[3] Kilbas A.A., Srivastava H.M., Trujillo J.J., *Theory and Applications of Fractional Differential Equations*, North-Holland Mathematics Studies, Vol. 204, Elsevier, Amsterdam (2006).
[4] Lubich, C., *Discretized fractional calculus*, SIAM J. Math. Anal. 17(3), 704-719 (1986).
[5] Matignon D., *Stability results for fractional differential equations with applications to control processing*, Computational Eng. in Sys. Appl., Vol. 2, Lille, France 963 (1996).
[6] Mickens R. E., *Difference equation models of differential equations having zero local truncation errors*, Dieferential Equations, North-Holland, Amsterdam (1984), 445-449.
[7] Mickens R. E., *Exact solutions to difference equation models of Burger's equation*, Numerical Methods for Partial Differential Equations 2 (1986).
[8] Mickens R. E., *Nonstandard Finite Difference Models of Differential Equations*, World Scientific, Singapore, (1994).

On the oscillation of higher order half linear delay dynamic equations

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Abstract

In this paper, we study sufficient conditions for the oscillatory and asymptotic behavior of the following higher-order half-linear delay dynamic equation

$$\left[p(t) + \left((x(t) + q(t)\,x(\tau(t)))^{\Delta^{m-1}}\right)^{\alpha}\right]^{\Delta} + r(t)\,x^{\beta}(\psi(t)) = 0, \qquad t \geq t_0$$

where we assume $\int\limits_{t_0}^{\infty} \frac{1}{p^{\frac{1}{\alpha}}(t)} \Delta t < \infty$. The main theorem of this paper improves some previously obtained results and thus presents a new approach.
This talk is based on a joint work with Veysel Fuat Hatipoğlu.

References

[1] C. Zhang, T. Li, B. Sun, E. Thandapani, On the oscillation of higher-order half-linear delay differential equations, *Applied mathematics letters*, Vol. 24, (2011), pp. 1618–1621.

[2] D. Uçar, Y. Bolat, Oscillatory behaviour of a higher order dynamic equation, *Journal of inequalities and applications*, Vol. 52, (2013).

[3] M. Bohner, A. Peterson, *Dynamic equations on time scales, an introduction with applications*, Birkhauser, Boston, 2001.

[4] R.P. Agarwal, M. Bohner, Basic Calculus on Time Scales and Some of its Applications, *Results Math.*, Vol. 35, (1999), pp. 3–22.

[5] R.P. Agarwal, E. Akın, S. Sun, Oscillation criteria for fourth-order nonlinear dynamic equations, *Communications on applied nonlinear analysis*, Vol. 18, (2011).

[6] S.R. Grace, On the oscillation of higher-order dynamic equations, *J. of advanced research*, Vol. 4, (2013), pp. 201–204.

[7] S.R. Grace, R.P. Agarwal, A. Zafer, Oscillation of higher order nonlinear dynamic equations on time scales, *Advances in difference equations*, Vol. 67, (2012).

[8] Y. Bolat, J.O. Alzabut, On the oscillation of higher-order half-linear delay difference equations, *Applied mathematics &information sciences*, Vol. 6, No. 3 (2012), pp. 423–427.

[9] Z. Han, S. Sun, C. Zhang, T. Li, Oscillation criteria of second-order nonlinear delay dynamic equations on time scales, *Proceedings of the 8th world congress on intelligent control and automation*, 2010, pp. 5762–5766.

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